Arc length $= \frac{x^\circ}{360}(2\pi r)$

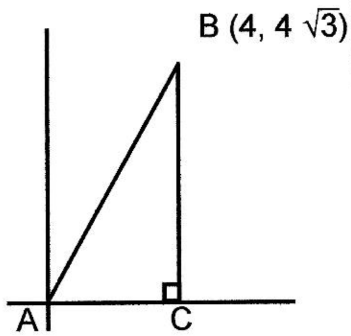
For a 30 degree angle in the **unit** circle, Arc length $= \frac{30^\circ}{360}(2\pi(1)) = \frac{\pi}{6}$ radians

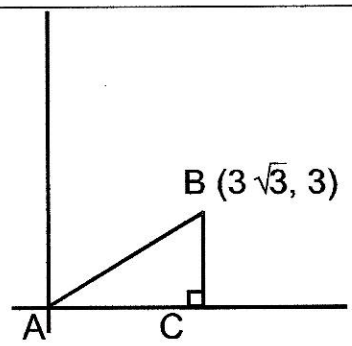
Introduction to Trigonometric Ratios with Special Right Triangles

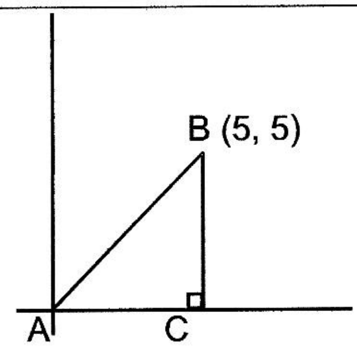
$\sin\theta = \frac{o}{h}$ $\cos\theta = \frac{a}{h}$ $\tan\theta = \frac{o}{a}$ $\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

where "*o*" *is* "opposite leg", "*a*" *is* "adjacent leg", *and* "*h*" *is* "hypotenuse"

1. Find the measure of each angle. Then, find the exact value of the sine, cosine, tangent, secant, cosecant, and cotangent for each angle. Rationalize all denominators and simplify completely.

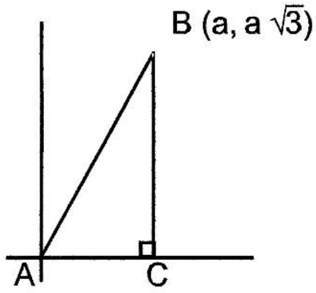
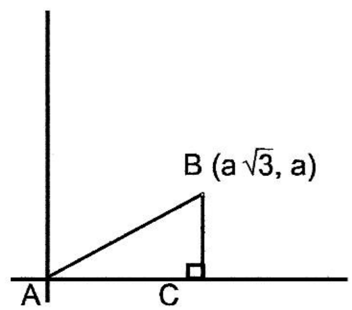
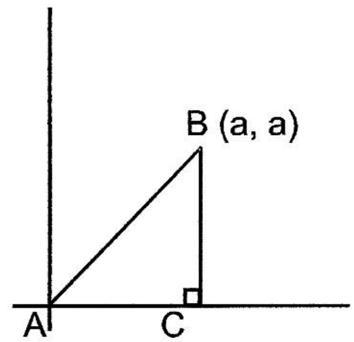






(a) m∠BAC = ______	(a) m∠BAC = ______	(a) m∠BAC = ______
(b) sin ______ = ______	(b) sin ______ = ______	(b) sin ______ = ______
(c) cos ______ = ______	(c) cos ______ = ______	(c) cos ______ = ______
(d) tan ______ = ______	(d) tan ______ = ______	(d) tan ______ = ______
(e) csc ______ = ______	(e) csc ______ = ______	(e) csc ______ = ______
(f) sec ______ = ______	(f) sec ______ = ______	(f) sec ______ = ______
(g) cot ______ = ______	(g) cot ______ = ______	(g) cot ______ = ______
(h) m∠ABC = ______	(h) m∠ABC = ______	(h) m∠ABC = ______
(i) sin ______ = ______	(i) sin ______ = ______	(i) sin ______ = ______
(j) cos ______ = ______	(j) cos ______ = ______	(j) cos ______ = ______
(k) tan ______ = ______	(k) tan ______ = ______	(k) tan ______ = ______
(l) csc ______ = ______	(l) csc ______ = ______	(l) csc ______ = ______
(m) sec ______ = ______	(m) sec ______ = ______	(m) sec ______ = ______
(n) cot ______ = ______	(n) cot ______ = ______	(n) cot ______ = ______
(o) AB = ________	(o) AB = ________	(o) AB = ________

2. Find the measure of each angle. Then, find the exact value of the sine, cosine, tangent, secant, cosecant, and cotangent for each angle. Rationalize all denominators; do not use double denominators.

		
(a) $m\angle BAC =$ ____	(a) $m\angle BAC =$ ____	(a) $m\angle BAC =$ ____
(b) sin ____ = ____	(b) sin ____ = ____	(b) sin ____ = ____
(c) cos ____ = ____	(c) cos ____ = ____	(c) cos ____ = ____
(d) tan ____ = ____	(d) tan ____ = ____	(d) tan ____ = ____
(e) csc ____ = ____	(e) csc ____ = ____	(e) csc ____ = ____
(f) sec ____ = ____	(f) sec ____ = ____	(f) sec ____ = ____
(g) cot ____ = ____	(g) cot ____ = ____	(g) cot ____ = ____

(h) What is the sine of 30 degrees regardless of the side length? What is the cosine of 60 degrees regardless of the side length? How does the sine of 30° compare to the cosine of 60°?

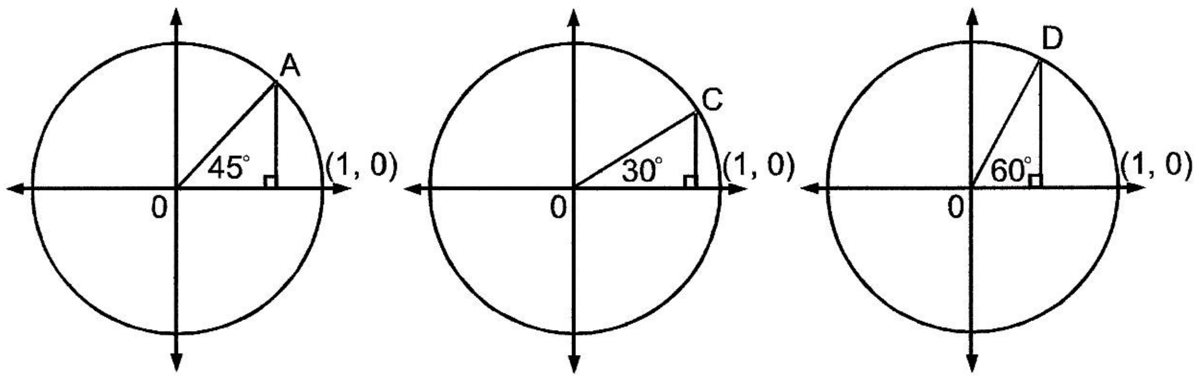
(i) Look at a trigonometric table and compare the values of sine of 43° and cosine of 47°. What is their relationship? Compare the values of the sine of 80° and the cosine of 10°. What is their relationship? What conclusion can you make about the sine of the angle and the cosine of its complement?

(j) Determine the value of the sine of 30° divided by the cosine of 30°. Compare the value to the tangent of 30°. Determine the value of the sine of 60° divided by the cosine of 60°. Compare the value to the tangent of 60°. Determine the value of the sine of 27° divided by the cosine of 27°. Compare the value to the tangent of 27°. What conclusion can you make about the relationship between the tangent of an angle and the sine of the same angle divided by the cosine of the same angle. Write the relationship as a formula.

(k) For which angles between 0 and 90 degrees is the tangent of the angle less than 1? Explain how you can justify your answer without consulting a table of values or a calculator. How large can values of the tangent be for an angle between 0 and 90 degrees? Can there be two angles in the same triangle with the same tangent? Justify your answers with reference to the tangent ratio in a right triangle ABC with legs a and b and hypotenuse c.

(l) Choose several angle measures x between 0 and 90 degrees. Using a calculator to determine the tangent of each angle x and the tangent of the complement. Find a relationship between these two tangents. State this relationship as an equation. Make a general conjecture about the tangents of an angle and its complement. Prove the conjecture by referring to a right triangle with legs a and b and hypotenuse c and one acute angle measuring x degrees.

3. Refer to the following diagrams:



(a) Find the coordinates of A, C, and D.

(b) Find the slope of $\overline{AO}$.

(c) Find the tangent of 45 degrees.

(d) Find the slope of $\overline{CO}$.

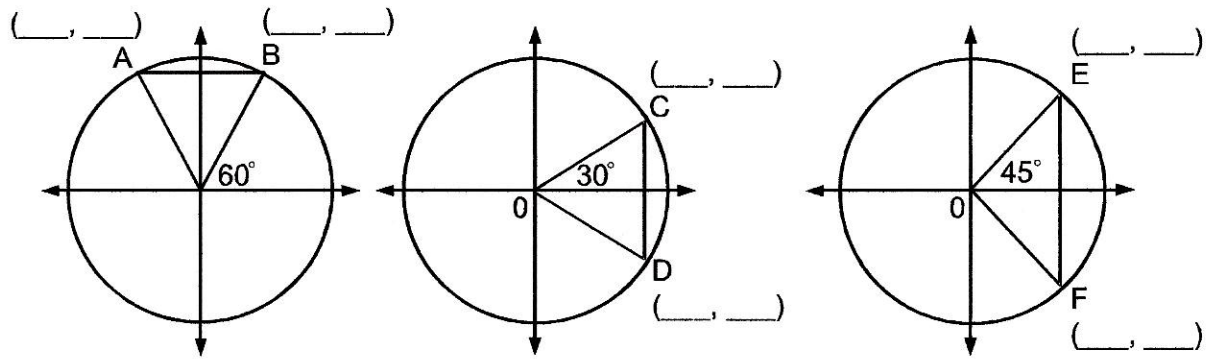
(e) Find the tangent of 30 degrees.

(f) Find the slope of $\overline{DO}$.

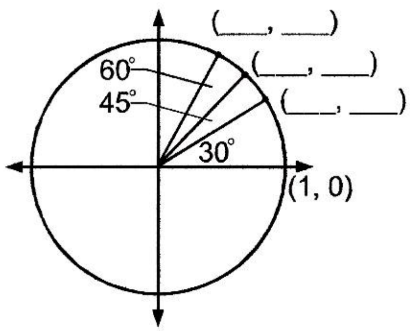
(g) Find the tangent of 60 degrees.

(h) Describe the slope in terms of a trigonometric ratio.

4. Find the coordinates of *A*, *B*, *C*, *D*, *E*, and *F*.



5. Put all the information in exercise 4 into one circle below. This circle is called the "unit circle". Answer the following for $0 \le \theta \le 90^\circ$



(a) Write the x value in the unit circle in terms of θ, the measure of the central angle.

(b) Write the y value in the unit circle in terms of θ, the measure of the central angle.

(c) As the angles become larger, what happens to the x value?
As the angles become larger, what happens to the y value?

(d) How small can x become? As x approaches this limit, what value does the y value approach?

(e) As x approaches zero, what angle is being approached?

(f) What is the value of $\cos 90^\circ$?

(g) What is the value of $\sin 90^\circ$?

(h) As θ becomes smaller, what happens to the x value?
As θ becomes smaller, what happens to the y value?

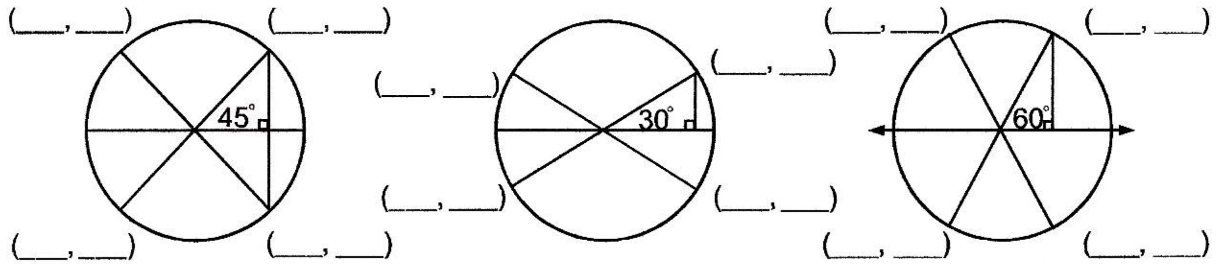
(i) How small can y become? As y approaches this value, what value does the x value approach?

(j) As y approaches zero, what angle is being approached?

(k) What is the value of $\cos 0^\circ$?

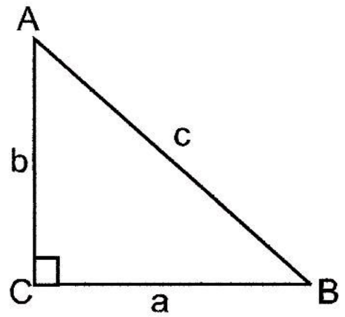
(l) What is the value of $\sin 0^\circ$?

6. Use the unit circles below to determine the following. Give the exact answers for the coordinates of the points below using special right triangle ratios.



$\sin 30^\circ$ = ______	$\cos 30^\circ$ = ______	$\tan 30^\circ$ = ______
$\sin 45^\circ$ = ______	$\cos 45^\circ$ = ______	$\tan 45^\circ$ = ______
$\sin 60^\circ$ = ______	$\cos 60^\circ$ = ______	$\tan 60^\circ$ = ______
$\sin 90^\circ$ = ______	$\cos 90^\circ$ = ______	$\tan 90^\circ$ = ______
$\sin 120^\circ$ = ______	$\cos 120^\circ$ = ______	$\tan 120^\circ$ = ______
$\sin 135^\circ$ = ______	$\cos 135^\circ$ = ______	$\tan 135^\circ$ = ______
$\sin 150^\circ$ = ______	$\cos 150^\circ$ = ______	$\tan 150^\circ$ = ______
$\sin 180^\circ$ = ______	$\cos 180^\circ$ = ______	$\tan 180^\circ$ = ______
$\sin 210^\circ$ = ______	$\cos 210^\circ$ = ______	$\tan 210^\circ$ = ______
$\sin 225^\circ$ = ______	$\cos 225^\circ$ = ______	$\tan 225^\circ$ = ______
$\sin 240^\circ$ = ______	$\cos 240^\circ$ = ______	$\tan 240^\circ$ = ______
$\sin 270^\circ$ = ______	$\cos 270^\circ$ = ______	$\tan 270^\circ$ = ______
$\sin 300^\circ$ = ______	$\cos 300^\circ$ = ______	$\tan 300^\circ$ = ______
$\sin 315^\circ$ = ______	$\cos 315^\circ$ = ______	$\tan 315^\circ$ = ______
$\sin 330^\circ$ = ______	$\cos 330^\circ$ = ______	$\tan 330^\circ$ = ______
$\sin 360^\circ$ = ______	$\cos 360^\circ$ = ______	$\tan 360^\circ$ = ______
$\csc 30^\circ$ = ______	$\sec 30^\circ$ = ______	$\cot 30^\circ$ = ______
$\csc 45^\circ$ = ______	$\sec 45^\circ$ = ______	$\cot 45^\circ$ = ______
$\csc 60^\circ$ = ______	$\sec 60^\circ$ = ______	$\cot 60^\circ$ = ______

7. Use triangle *ABC* to determine the following



(a) $a = 5$, $b = 5$, $m\angle A =$ ____ ; $m\angle B =$ ____ ; $c=$ __________

(b) $a = 2$, $b = 2\sqrt{3}$, $m\angle A =$ ____ ; $m\angle B =$ ____ ; c= __________

(c) $a = 3\sqrt{2}$, c = 6, $m\angle A =$ ____ ; $m\angle B =$ ____ ; $b=$ __________

(d) $b = 5$, $c = 10$, $m\angle A =$ ____ ; $m\angle B =$ ____ ; $a=$ __________

(e) $a = 3$, $b = 4$, $c =$ __________ ; $m\angle A =$ ____ ; $m\angle B =$ ____

(f) $a = 5$, $c = 13$, $c =$ __________ ; $m\angle A =$ ____ ; $m\angle B =$ ____

(g) $b = 15$, $c = 17$, $c =$ __________ ; $m\angle A =$ ____ ; $m\angle B =$ ____

(h) Create three more sets of primitive Pythagorean triples and find the missing angles.

8. Answer the following:

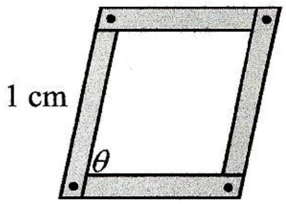
(a) Can a Primitive Pythagorean triple triangle be a special right triangle? Explain.

(b) In what type of right triangle would the legs and angles all be whole numbers?

(c) In what type of right triangle would the sides and angles all be whole numbers?

9. Complete the table below by determining the area of the rhombus and graph the coordinate answers.

(a)



m $\angle\theta$ in degrees	30	45	60	90	120	135	150
Area							

(b) Graph the information shown in the table.

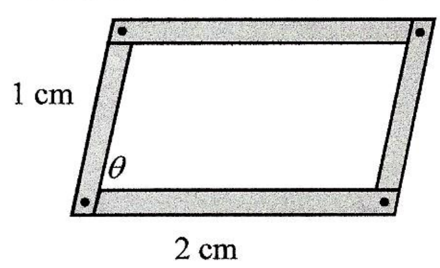
(c) Name the trig ratio that could be used to determine the height. The graph above represents one piece of a _______ wave.

(d) Use the values found on the unit circle to extend your graph.

(e) Use your graphing calculator to sketch a graph of the sine function [0, 360]. Trace to a value in the table above and compare your answer. Trace to a point other than those listed in the table above and discuss its meaning.

(f) Write the equation for the area in terms of sine.

(g) Complete the table below by determining the area of the parallelogram.



m $\angle\theta$ in degrees	30	45	60	90	120	135	150
Area							

(h) Sketch the graph in dashed lines on the grid in part (b).

(i) Write the equation for area in terms of sine for the table above.

(j) How are the two graphs related?